Angular Segmentation (Sn) method symmetric discrete directions set [1]

Number of directions in an octant is

$$M/8 = n(n+1)/2 . \quad (1)$$

<u>For directions coordinates</u>

$$\mu_j^2 + \mu_k^2 + \mu_l^2 = 1,\ j,k,l = 1,\dots,n \text{ and} \quad (2)$$

$$j + k + l = n + 2,\ j,k,l = 1,\dots,n . \quad (3)$$

Implying squared coordinates linearity on its number

$$\mu_j^2 = p + q \cdot j , \quad (4)$$

this linear dependency coefficients p and q depending on the μ_1^2 and n may be obtained.

<u>For quadrature weights</u>

$$\int_{4\pi} f(\vec{\Omega}) d\vec{\Omega} \approx 4\pi \sum_{d=1}^{M} C_d f(\vec{\Omega}_d), \quad (5)$$

and therefore

$$\sum_{d=1}^{M} C_d \left(\vec{i}_\gamma \cdot \vec{\Omega}_d\right)^m = \frac{1}{m+1},\ \forall \gamma \quad (6)$$

due to octants similarity

$$8 \sum_{d=1}^{M/8} C_d \Omega_{d\gamma}^m = \frac{1}{m+1} . \quad (7)$$

Assuming point with location vector $\vec{i}_\gamma$ to be pole directions in an octant may be ordered by azimuth and polar angles with their numbers presented as

$$d = j + (2n+2-k)(k-1)/2 , (8)$$

where j denotes azimuth angle φ and k denotes polar angle θ. Thus the equation (7) may be rewritten

$$8 \sum_{k=1}^{n} \sum_{j=1}^{n+1-k} C_{j+(2n+2-k)(k-1)/2} \Omega^m_{(j+(2n+2-k)(k-1)/2)\gamma} = \frac{1}{m+1} . \quad (9)$$

Due to angular nodes construction their coordinates with the same $k = 1,\dots,n$ are the same

$$\Omega_{1+(2n+2-k)(k-1)/2\,\gamma} = \dots = \Omega_{nk-(k-1)k/2\,\gamma} = \mu_k . \quad (10)$$

By denoting

$$w_k = 8 \sum_{j=1}^{n+1-k} C_{j+(2n+2-k)(k-1)/2} \quad (11)$$

the equation (9) left-hand side may be rewritten

$$8 \sum_{k=1}^{n} \sum_{j=1}^{n+1-k} C_{j+(2n+2-k)(k-1)/2} \Omega^m_{(j+(2n+2-k)(k-1)/2)\gamma} = \sum_{k=1}^{n} w_k \mu_k^m . (12)$$

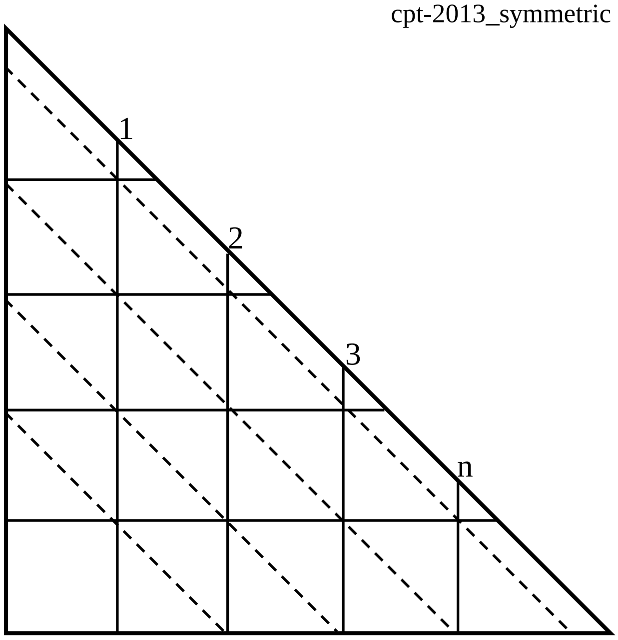
That finally leads to two systems of the $(n+1)$ non-linear equations, the odd-even moments system

$$\sum_{k=1}^{n} w_k \mu_k^m = \frac{1}{m+1},\ m = 0,\dots,n \quad (13)$$

or the even-only moments system

$$\sum_{k=1}^{n} w_k \mu_k^{2m} = \frac{1}{2m+1},\ m = 0,\dots,n \quad (14)$$

from which the polar weights w_k and the first ordinate



S22

11B 12A 139 148 157 166 175 184 193 1A2 1B1
21A 229 238 247 256 265 274 283 292 2A1
319 328 337 346 355 364 373 382 391
418 427 436 445 454 463 472 481
517 526 535 544 553 562 571
616 625 634 643 652 661
715 724 733 742 751
814 823 832 841
913 922 931
A12 A21
B11

S2

111

S20

11A 129 138 147 156 165 174 183 192 1A1
219 228 237 246 255 264 273 282 291
318 327 336 345 354 363 372 381
417 426 435 444 453 462 471
516 525 534 543 552 561
615 624 633 642 651
714 723 732 741
813 822 831
912 921
A11

S4

112 121
211

S18

119 128 137 146 155 164 173 182 191
218 227 236 245 254 263 272 281
317 326 335 344 353 362 371
416 425 434 443 452 461
515 524 533 542 551
614 623 632 641
713 722 731
812 821
911

μ_1 or its square μ_1^2 may be obtained.

Quadrature nodes weights may be assumed to be the sums of the axial weights set $\{a_l \mid l = 1,\dots,n\}$ that agreed with the nodes coordinates set $\{\mu_l \mid l = 1,\dots,n\}$

$$C_{j+(2n+2-k)(k-1)/2} = a_j + a_k + a_{n+2-k-j}\,. \qquad (15)$$

The layouts produced by that rule for different n are listed on the right. Due to in-octant symmetry the distinct quadrature weights (on the right they are colored in teal) number

$$N = \sum_{k=1}^{n} \sum_{j=1}^{(n+2-k)/2} 1\,. \qquad (16)$$

Thus the $(n-1)$ linear equations (11) for $k = 2,\dots,n$ (due to the equations (13) or (14) for $m = 0$ the all n equations (11) are not independent) and the N independent linear equations (15) form the linear equations system in the N distinct node weights C_d and n axis weights a_l from which the system of N independent linear equations in N distinct node weights C_d may be derived by eliminating all a_l variables. From the last system the distinct node weights may be obtained.

Two issues are essential to be noted. The first one is that "n" in quadrature sets nomenclature "Sn" is twice greater than n (number of in-octant coordinates along an axis) in formulæ above. The second one is that above described approach leads to negative polar weights in the solutions of the equations systems (13) for $n > 4$ and (14) for $n > 11$, and to negative nodal weights for $n = 9$ and $n = 11$.

S6

113 122 131
212 221
311

S8

114 123 132 141
213 222 231
312 321
411

S10

115 124 133 142 151
214 223 232 241
313 322 331
412 421
511

S12

116 125 134 143 152 161
215 224 233 242 251
314 323 332 341
413 422 431
512 521
611

S14

117 126 135 144 153 162 171
216 225 234 243 252 261
315 324 333 342 351
414 423 432 441
513 522 531
612 621
711

S16

118 127 136 145 154 163 172 181
217 226 235 244 253 262 271
316 325 334 343 352 361
415 424 433 442 451
514 523 532 541
613 622 631
712 721
811

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Reference

1. K. D. Lathrop, B. G. Carlson "Discrete Ordinates Angular Quadrature of the Neutron Transport Equation" Los Alamos Scientific Laboratory Technical Report, LA-3186, September, 1964. DOI:10.2172/4666281

Document history

December 2013	Produced
July 2015	Emendated

Realisation

The "Mathematica" environment script that implements above described approach could be found at

http://cpt.imamod.ru/alg/as/symmetric/AS_symmetric_set.txt

The resulting sets could be downloaded at

In-octant sets

http://cpt.imamod.ru/alg/as/symmetric/sets/0,125/S4.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,125/S6.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,125/S8.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,125/S10.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,125/S12.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,125/S14.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,125/S16.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,125/S20.TXT

Full sets

http://cpt.imamod.ru/alg/as/symmetric/sets/full/S4_XYZ.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/full/S6_XYZ.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/full/S8_XYZ.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/full/S10_XYZ.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/full/S12_XYZ.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/full/S14_XYZ.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/full/S16_XYZ.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/full/S20_XYZ.TXT

X-hemisphere sets

http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/x/S4+X.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/x/S6+X.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/x/S8+X.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/x/S10+X.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/x/S12+X.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/x/S14+X.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/x/S16+X.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/x/S20+X.TXT

Y-hemisphere sets

http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/y/S4+Y.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/y/S6+Y.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/y/S8+Y.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/y/S10+Y.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/y/S12+Y.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/y/S14+Y.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/y/S16+Y.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/y/S20+Y.TXT

Z-hemisphere sets

http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/z/S4+Z.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/z/S6+Z.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/z/S8+Z.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/z/S10+Z.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/z/S12+Z.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/z/S14+Z.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/z/S16+Z.TXT
http://cpt.imamod.ru/alg/as/symmetric/sets/0,5/z/S20+Z.TXT

Sets stored in TSV (tab-separated value) tabular data format with $\cos(\theta_d)$, φ_d, and $8C_d$ (polar angle cosine, azimuth angle, and multiplied by eight node weight) columns.

This note stored at

http://cpt.imamod.ru/alg/as/symmetric/cpt-2013_symmetric.pdf